Practice questions

1) Usain Bolt runs 100m ± 2cm in 9.63 ± 0.005 s. Calculate his average speed and its absolute uncertainty.

$$s = \frac{d}{t} = \frac{100}{9.63} = 10.384\ldots\ \text{ms}^{-1}$$

% uncertainty in d: $\frac{2\times10^{-2}}{100} \times 100\% = 2.00 \times 10^{-2}\ \%$

% uncertainty in t: $\frac{5\times10^{-3}}{9.63} \times 100\% = 5.19 \times 10^{-2}\ \%$

Total % uncertainty: $7.19 \times 10^{-2}\ \%$

Total absolute uncertainty: $10.384\ldots \times \frac{7.19\times10^{-2}}{100} = 7.47 \times 10^{-3}\ \text{ms}^{-1}$

Final answer: $\boldsymbol{s = 10.384 \pm 0.007\text{ms}^{-1}}$

2) Richard Feynman stands on a set of bathroom scales. They read 83 kg. His daughter, Michelle, runs into the bathroom and he picks her up. The scales now read 105 kg. If the scales have an uncertainty of 0.5 kg, calculate Michelle's mass and its absolute uncertainty.

$\Delta m = 105 - 83 = 22$ kg

Total absolute uncertainty: $2 \times 0.5 = 1$ kg

Final answer: $\boldsymbol{m = 22 \pm 1}$ **kg**

3) Newton's constant of gravitation (abbreviated to *G*) is a fundamental constant which describes how gravity behaves. Our current (2010) best value is

$G = 6.67384 \pm 0.00080 \times 10^{-11}\ \text{Nm}^2\text{kg}^{-2}$.

Our current best value for the charge on an electron is:

$e = 1.602\,176\,565 \pm 0.000\,000\,035 \times 10^{-19}$ Coulombs.

Which of these constants would you describe as the most accurately known? Use calculations to justify your reasons.

% uncertainty in G $= \frac{0.0008}{6.67384} \times 100\% = 0.012\%$

% uncertainty in *e* $= \frac{0.000\,000\,035}{1.602\,176\,565} \times 100\% = 2.2 \times 10^{-6}\ \%$

Therefore, *e* is considerably (about $\frac{0.012}{2.2\times10^{-6}} \approx 5500$ times) more accurately known.

4) A packet of printer paper contains 500 sheets of paper. Its height is 52 ± 0.5 mm, and its mass is 2.521 ± 0.001 kg. Calculate:

a) The thickness of one piece of the paper, including its uncertainty

$$\frac{52}{500} = 0.104 \text{ mm}$$

Uncertainty: $\frac{0.5}{500} = 1 \times 10^{-3}$ mm

Final answer: $\boldsymbol{t = 0.104 \pm 0.001}$ **mm** **(or 0.96%)**

b) The mass of one piece of paper, including its uncertainty

$$\frac{2.521}{500} = 5.042 \times 10^{-3} \text{ kg}$$

Uncertainty: $\frac{0.001}{500} = 2 \times 10^{-6}$ kg

Final answer: $\boldsymbol{m = 5.042 \pm 0.002}$ **g** **(or 0.04%)**

c) The packet says each sheet of paper measures 210 mm by 297 mm. If both of these measurements are ± 0.5 mm, calculate the Volume of one sheet of paper, including its uncertainty.

$V = 210 \times 297 \times 0.104 = 6486 \ldots$ mm^3

Total % uncertainty: $\left(\frac{0.5}{210} + \frac{0.5}{297} + \frac{0.001}{0.104}\right) \times 100\% = 1.4\%$

Total absolute uncertainty: $6490 \times 1.4\% = 89$ mm^3

Final answer: $\boldsymbol{V = 6490 \pm 90}$ **mm^3**

d) Using your answers to b) and c), calculate the Density of the paper, including its uncertainty

$$\rho = \frac{m}{V} = \frac{5.042}{6490} = 7.77 \times 10^{-4} \text{ g mm}^{-3}$$

Total % uncertainty: $\left(\frac{2\times10^{-6}}{5.042} \times 100\%\right) + 1.4\% = 1.4\%$ (!)

Total absolute uncertainty: $7.77 \times 1.4\% = 0.11$ g mm^{-3}

Final answer: $\boldsymbol{\rho = 7.8 \pm 0.1 \times 10^{-4}}$ **g mm^{-3}**

5) A vacuum bazooka fires a ball horizontally at a height of 1.115 ± 0.01 m above the floor. It strikes a board placed 9.5 ± 0.02 m away. It hits the board at a height of 0.71 ± 0.005 m above the floor. Using $g = 9.81$ ms^{-2} and ignoring air resistance, calculate:

a) The height it falls during its flight, and its uncertainty

$1.115 - 0.71 = 0.405$ m

Uncertainty: 0.01 + 0.005 = 0.015 m

Final answer: $\boldsymbol{\Delta h = 0.405 \pm 0.015}$ **m** (or 3.7%)

b) The time it takes to hit the board from leaving the gun, and its uncertainty

$s = ut + \frac{1}{2}at^2$

$$t = \sqrt{\frac{0.405 \times 2}{9.81}} = 0.287 \text{ s}$$

Uncertainty $= 3.7 \times \frac{1}{2} = 1.85\%$

Absolute uncertainty $= 0.287 \times 1.85\% = 0.005$ s

Final answer: $\boldsymbol{t = 0.287 \pm 0.005}$ **s** (or 1.9%)

c) Its muzzle velocity (*i.e.* the speed the ball leaves the gun at), and its uncertainty

$$v = \frac{s}{t} = \frac{9.5}{0.287} = 33.1 \text{ ms}^{-1}$$

Uncertainty $= 1.85\% + \frac{0.02}{9.5} = 2.1\%$

Absolute uncertainty $= 33.1 \times 2.1\% = 0.7 \text{ ms}^{-1}$

Final answer: $\boldsymbol{v = 33.1 \pm 0.7 \text{ ms}^{-1}}$ (or 2.1%)