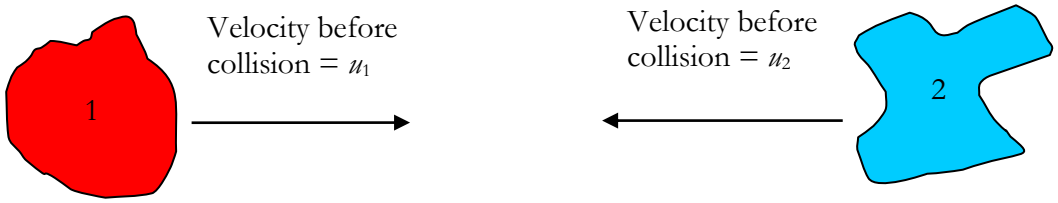
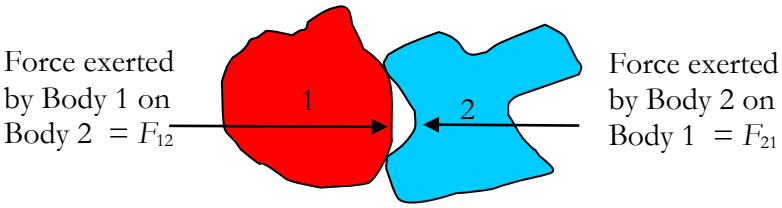
Deriving Conservation of Momentum from N(III)

Consider a collision between bodies 1 and 2:

Before collision:



During collision:



After collision:

N(III) states that, during the collision:

$$F_{12} = -F_{21}$$

Since both forces act for the same time during the collision:

$$F_{12}t = -F_{21}t$$

N(II) states that $Ft = mv - mu$, so:

$$m_1v_1 - m_1u_1 = -(m_2v_2 - m_2u_2)$$

i.e. the change in each body's momentum is equal and opposite to the change in the other body's momentum. Taking out the brackets on the right-hand side of the above equation gives:

$$m_1v_1 - m_1u_1 = -m_2v_2 + m_2u_2$$

Group the expressions involving u and v on either side of the equation:

$$-m_1u_1 - m_2u_2 = -m_1v_1 - m_2v_2$$

Inverting the sign of both sides gives:

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

In other words: **Total momentum before the collision = Total momentum after the collision**