Finding the Mass of Pluto

Pluto has a satellite (moon) known as Charon, which is seen to orbit Pluto with a period of 6.39 days.

From earth, when Pluto is at a distance of 7.39×10^9 km, the orbit of the satellite subtends an angle (appears to have an angular diameter) of 1.06 seconds of arc.

There are 3600 seconds of arc in a degree, and $\frac{2\pi}{360}$ degrees in a radian.

$G = 6.67 \times 10^{-11}$ Nm^2kg^{-2}

From this information alone, calculate the mass of Pluto.

Solution

Gravity provides centripetal force for orbit:

$$\frac{GMm}{r^2} = \frac{mv^2}{r}$$

Which simplifies to:

$$\frac{GM}{r} = v^2$$

Or:

$$\boxed{M = \frac{v^2 r}{G}}$$

We already know G, so need values for r and v.

We can only get v from $v = \frac{d}{t} = \frac{2\pi r}{t}$ for the orbit, so we must find r first.

Finding *r*

[diagram of orbit being observed from Earth]

$\sin\theta = \frac{2r}{7.39\times10^{12}}$ and $\sin\theta \approx \theta$ if θ is small and measured in radians.

$$1.06'' = \frac{1.06}{60^2} = 2.94\times10^{-4}\text{ degrees} = 2.94\times10^{-4}\times\frac{2\pi}{360} = 5.14\times10^{-6}\text{ radians}$$

$$r = \frac{\theta\times7.39\times10^{12}}{2} = \frac{5.14\times10^{-6}\times7.39\times10^{12}}{2} = 1.90\times10^{7}\text{ metres}$$

Finding *v*

$$v = \frac{d}{t} = \frac{2\pi r}{t} = \frac{2\pi\times1.90\times10^{7}}{6.39\times24\times60^2} = 216\,\text{ms}^{-1}$$

Final calculation:

$$\boxed{M = \frac{v^2 r}{G} = \frac{216^2\times1.90\times10^{6}}{6.67\times10^{-11}} = 1.33\times10^{22}\text{kg}}$$

[*Textbooks give more like 1.30 x 10^{22} kg; the above calculation assumes Pluto's centre is the centre of mass of the orbit which it isn't. Pluto and Charon are so similar in mass, their common centre of mass is actually outside Pluto itself.*]