

## Dice Half-life Paradox?

In the popular GCSE/A-level practical where you roll dice to simulate the action of radioactive decay, you usually find that the 'half-life' of a set of 6-sided dice is about 4 throws. But how close is this to the theoretical 'right answer'? Surprisingly, you get two different answers depending on how you approach the problem:

### Proof I

Each time you throw  $N$  dice and remove the 6s, you will be left with  $\frac{5}{6}N$  dice. After  $n$  throws you have  $\frac{1}{2}N$  dice remaining. Therefore:

$$\left(\frac{5}{6}\right)^n = 0.5$$

Taking logs of both sides gives:

$$n \ln\left(\frac{5}{6}\right) = \ln(0.5)$$

So:

$$n = \frac{\ln(0.5)}{\ln\left(\frac{5}{6}\right)} = 3.801 \text{ throws}$$

### Proof II

Radioactivity theory ( $N = N_0 e^{-\lambda t}$ ) gives:

$$T_{\frac{1}{2}} = \frac{\ln 2}{\lambda}$$

Where  $\lambda$  = probability of decay =  $\frac{1}{6}$  for a single dice .

Therefore:

$$n = T_{\frac{1}{2}} = \frac{\ln 2}{\lambda} = 4.159 \text{ throws}$$

The two methods give noticeably different answers to the problem. Both cannot be simultaneously correct. Is this a genuine paradox, or are there some hidden assumptions which might be causing the apparent problem?