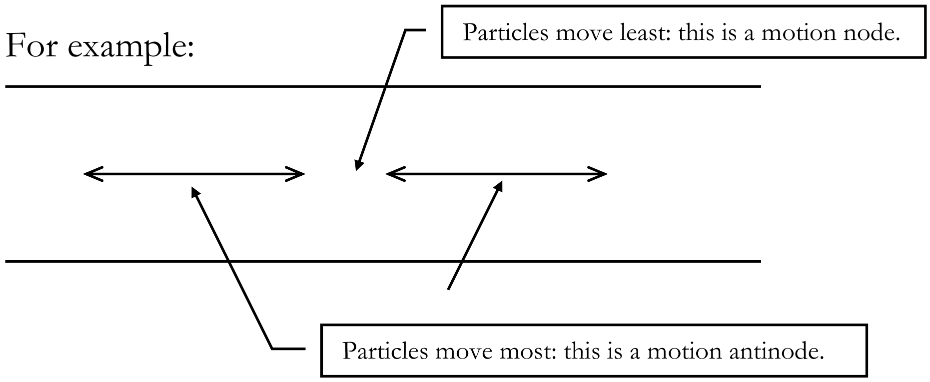
Standing Waves in Pipes

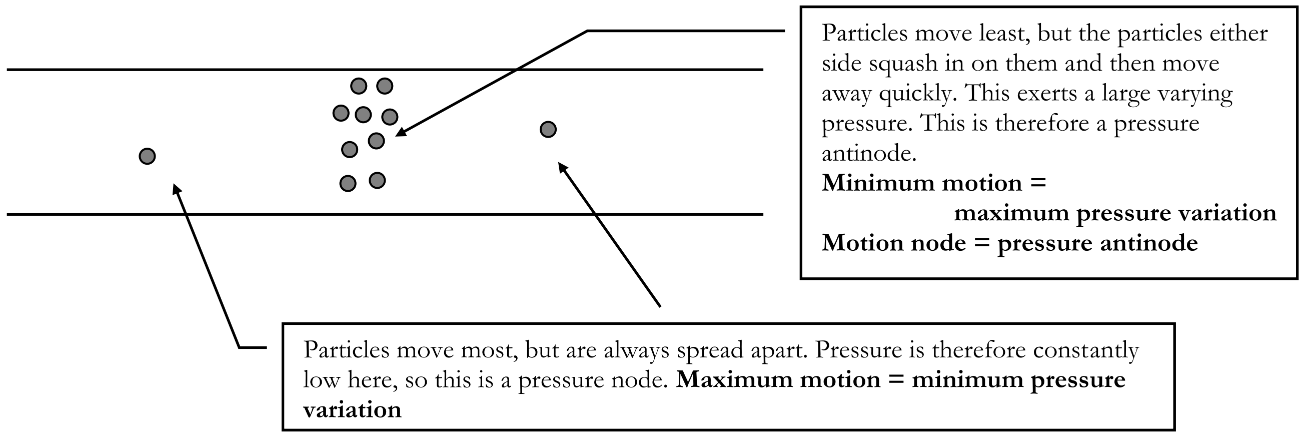
In any standing wave, you know there are **nodes** (where a certain quantity does not change) and **antinodes** (where the same quantity varies by a maximum amount compared to anywhere else on the standing wave). For standing waves on strings, it is obvious to show the motion of the string itself in a diagram, so a node is where the string does not move, and an antinode is where it moves most.

However, in a pipe, it is not as obvious what to show: do you show how the **pressure** in the pipe changes, or how the **motion of the air** changes? It doesn't really matter, but you must be perfectly clear about which you are showing, because a node for pressure is an antinode for motion and *vice versa*!

For example:



To show it another way, the particles in the pipe might look like this at a certain moment in time (drawn to exactly the same scale as the diagram above):

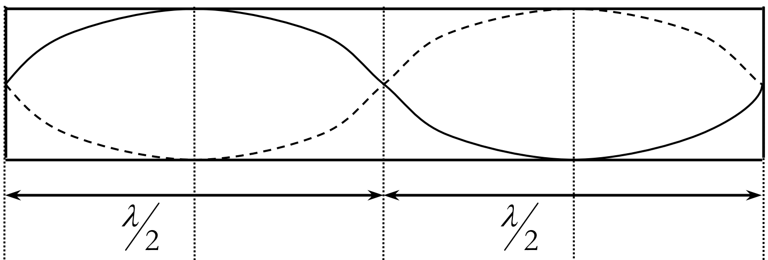


In the experiment with powder in a tube and a loudspeaker to create a standing wave (Kundt's Tube) the powder collects in areas of minimum particle motion (*i.e.* motion nodes/pressure antinodes).

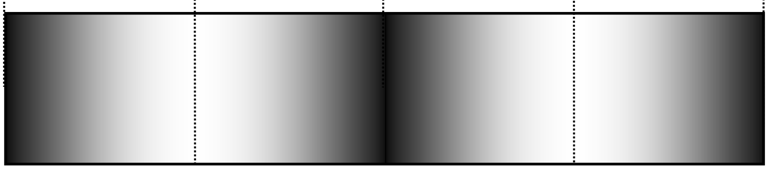
It's less usual, but you can also use shading to represent the motion of particles: where the particles are clumped together (*i.e.* a motion node/pressure antinode), you shade it **dark**.

However, it is easier in exams to represent the motion of air by curved sine wave lines, a bit like we use when drawing standing waves on strings. Again, though, how do you show <u>horizontal</u> motion of air with a sine wave shape with <u>vertical</u> amplitude?

It is perhaps most logical if we imagine particle motion as the *y*-axis amplitude of the wave. In other words, where the amplitude of the wave shape is **large**, the particles are moving **most** = **motion antinode** = **pressure node** = shaded **light**; and *vice versa.* Therefore:



…is what you can show by shading as:



Using the idea of a wave-shape also makes it easier to understand where the half- and quarter-wavelengths fit into the pipe, as shown above.

To sum up so far:

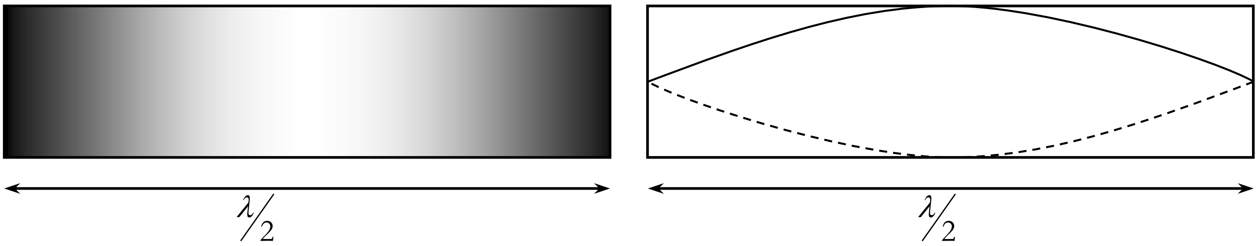
Amplitude of wave shape	**Particle motion**	**Motion…**	**Pressure…**	**Shading**
Large	Fastest	Antinode	Node	Light
Small	Least	Node	Antinode	Dark

Now we can use this to show how standing waves are different in closed, open and half-closed pipes…

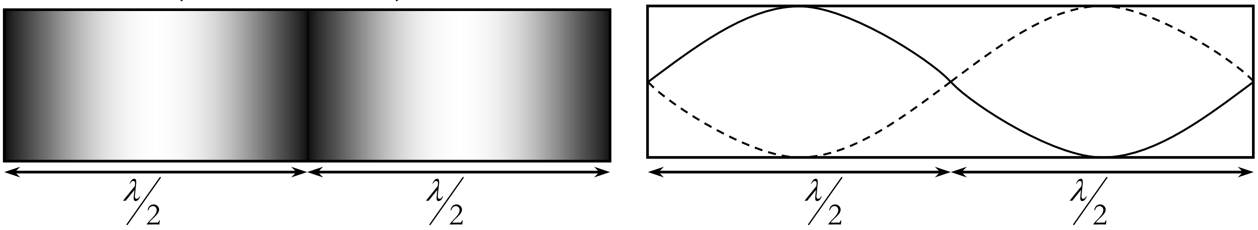
Standing waves in a pipe closed at both ends

The only simple rule you have to remember in this situation is that the air at both **closed** ends of the pipe **cannot move**, so you must always have a **motion node** at either end of the pipe. This should be fairly obvious! The rest of the diagram just comes from this rule:

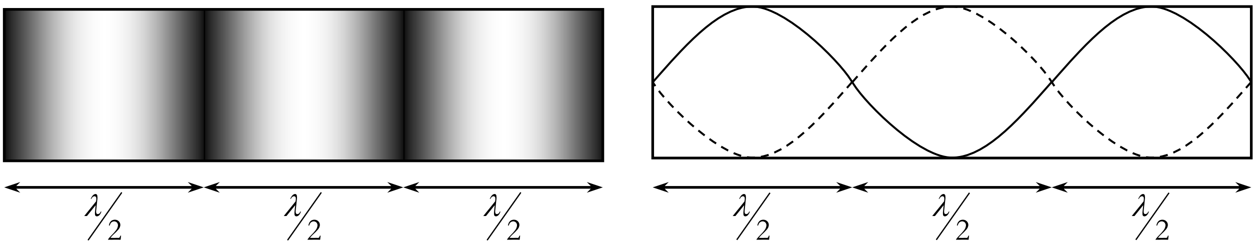
Fundamental (= 1st Harmonic):



1st Overtone (= 2nd Harmonic):



2nd Overtone (= 3rd Harmonic):



As you can see, you always get a **whole** number of **half**-wavelengths fitting into the tube, whatever the frequency of the standing wave.

Therefore: $\boxed{\text{Length of tube} = \frac{n\lambda}{2}}$, where $n = 1, 2, 3\ldots$ and λ = wavelength of the wave.

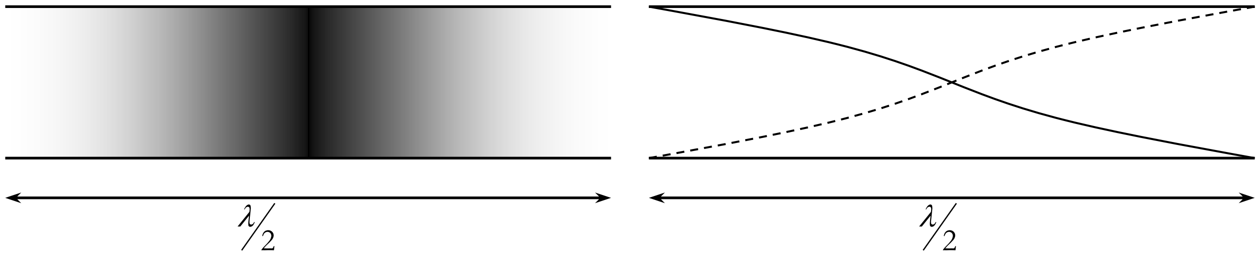
Since $f = {}^{c}/_{\lambda}$, the fundamental frequency must be given by $\boxed{f = \frac{c}{2L}}$, where L is the length of the tube and c is the speed of sound in the tube.

Then, since every overtone after the fundamental adds an extra half-wavelength within the tube, the frequency of the first overtone must be at $2f$, the second overtone at $3f$, the third at $4f$ and so on.

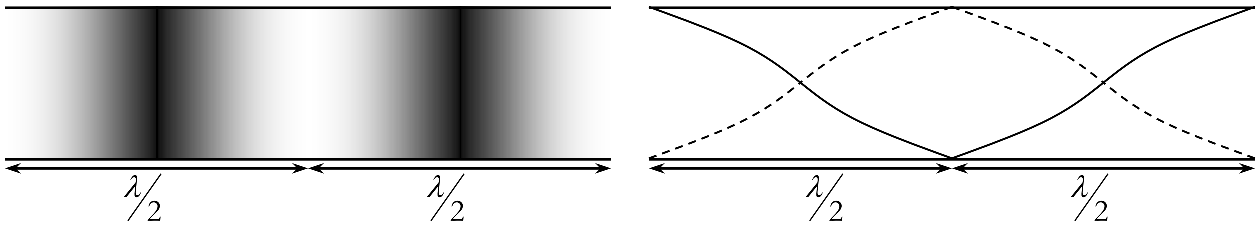
Standing waves in a pipe open at both ends

The only simple rule you have to remember in this situation is that the air at both **open** ends of the pipe **moves most**, so you must always have a **motion antinode** at either end of the pipe. This is just the opposite rule for a pipe closed at both ends, which should also be fairly obvious! The rest of the diagram just comes from this rule:

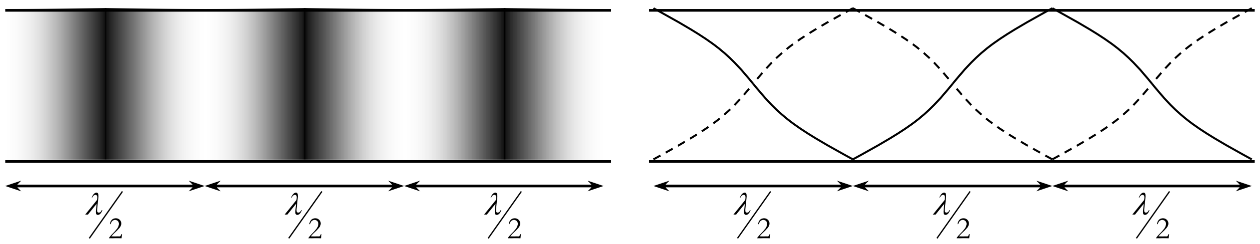
Fundamental (= 1st Harmonic):



1st Overtone (= 2nd Harmonic):



2nd Overtone (= 3rd Harmonic):



As you can see, you always get a **whole** number of **half**-wavelengths fitting into the tube, whatever the frequency of the standing wave.

Therefore: $\boxed{\text{Length of tube} = \frac{n\lambda}{2}}$, where $n = 1, 2, 3\ldots$ and λ = wavelength of the wave.

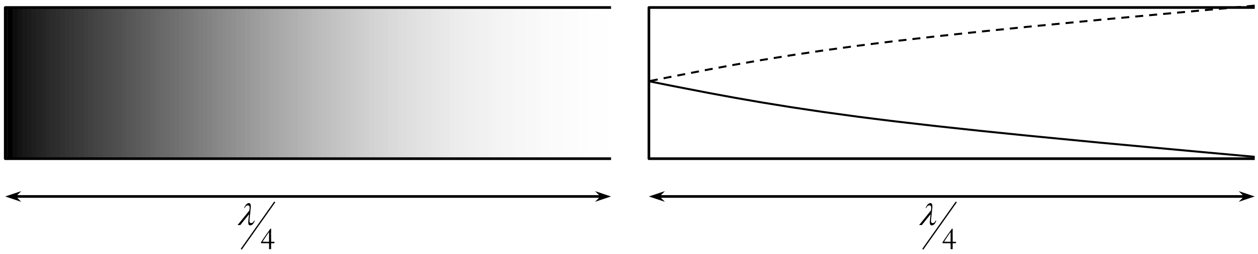
Since $f = {}^{c}\!/\!_{\lambda}$, the fundamental frequency must be given by $\boxed{f = \frac{c}{2L}}$, where L is the length of the tube and c is the speed of sound in the tube.

Then, since every overtone after the fundamental adds an extra half-wavelength within the tube, the frequency of the first overtone must be at $2f$, the second overtone at $3f$, the third at $4f$ and so on.

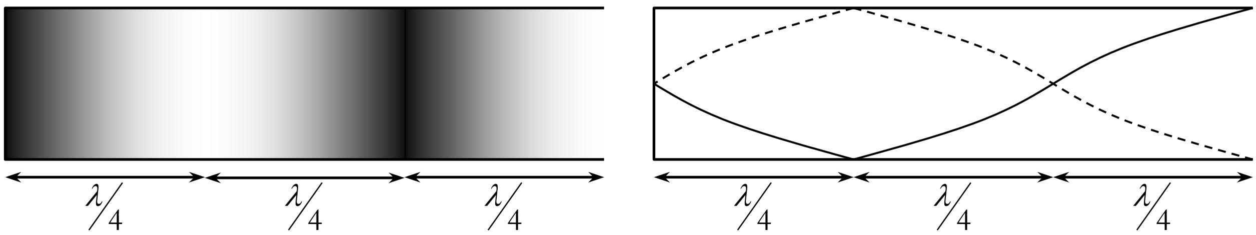
Standing waves in a tube open at one end

As you should be able to spot by now, the rules for pipes open at one end and closed at the other are simply a combination of the rules we have seen for completely closed and completely open pipes. In other words, the **closed** end must always be a **motion node**, the **open** end must always be a **motion antinode**. The diagrams therefore look like this:

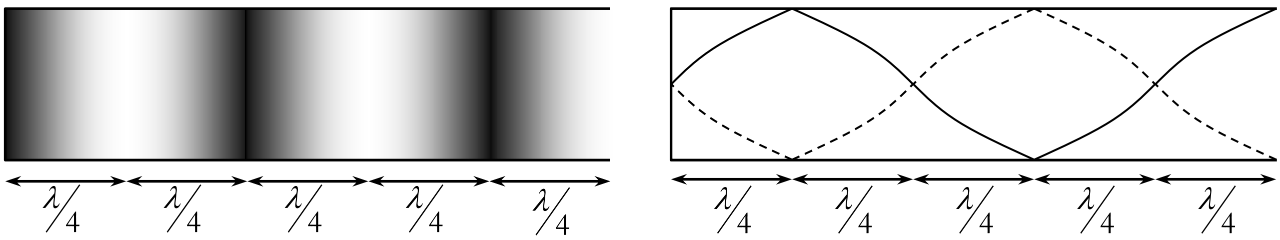
Fundamental (= 1st Harmonic):



1st Overtone (= 3rd Harmonic[1]):



2nd Overtone (= 5th Harmonic):



As you can see, with this type of tube, you always get an **odd** number of **quarter**-wavelengths fitting into the tube, whatever the frequency of the standing wave.

Therefore: $\boxed{\text{Length of tube} = \frac{(2n-1)\lambda}{4}}$, where n = 1, 2, 3… and λ = wavelength of the wave.

Since $f = c/\lambda$, the fundamental frequency must be given by $\boxed{f = \frac{c}{4L}}$, where L is the length of the tube and c is the speed of sound in the tube.

Then, since every overtone after the fundamental adds two extra quarter-wavelengths (= half a wavelength) within the tube, the frequency of the first overtone must occur at $3f$ (*i.e.* the 3rd harmonic), the second must be at $5f$ (= 5th harmonic), the third at $7f$ and so on.

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[1] A harmonic is any integer multiple of the fundamental frequency, whereas an overtone is any frequency higher than the fundamental which the system is actually capable of vibrating at. A harmonic need not be an overtone (*e.g.* in the tube above, the 2nd and 4th harmonics are not overtones), and an overtone need not be a harmonic (*e.g.* in drums). For more details, see hyperphysics.phy-astr.gsu.edu/hbase/music/otone.html

In real life, however…

… you don't get the motion antinode occurring exactly at the end of the pipe, but actually a few centimetres outside the end. This means you have to add a small correction factor (called the **end correction**) to any equations you use if you are making something practical (like a musical instrument) which needs to produce very precise frequencies. The exact size of the end correction can depend on lots of other factors (like the diameter of the pipe, which of course is not a constant in most wind instruments, as the tube flares outwards to make the 'bell'), which is one reason why making good quality wind instruments is neither easy nor cheap.

A general rule for calculating end corrections is that **End correction = 0.29 x Diameter of Tube**. Therefore, a bassoon would have a bigger end correction than an oboe because it has a bigger diameter[2]. End corrections also apply to **each** end of a tube, so, for example, a flute (which is basically a tube open at both ends) would have double the end correction of a tenor recorder (which is a similar diameter but is only open at one end).

The University of New South Wales in Australia has a very extensive website showing their research into the acoustics of different musical instruments: www.phys.unsw.edu.au/music/

[2] Which leads to the highly insulting musician's joke 'Why is a bassoon better than an oboe? Because it burns longer'.

Measuring End Corrections

Boomwhackers (yes, really) are hollow plastic tubes which make a musical note when hit gently. Longer boomwhackers produce *higher/lower* frequency notes because their length produces a *longer/shorter* wavelength standing wave in the tube. [*delete as applicable*]

Practical

Measure the boomwhacker tubes, and use the correct formula from earlier in the booklet to complete the following table. The shaded columns require you to measure the actual tubes, the remaining columns require you to calculate values.

Note	*f* (Hz)	Diameter of tube (cm)	Actual length (cm)	Theoretical length with zero end correction (cm)	End correction (cm)	End correction as fraction of diameter
C	261.6					
D	293.7					
E	329.6					
F	349.2					
G	392.0					
A	440.0					
B	493.9					
C	523.3					