Measuring the Hubble parameter and finding the Age of the Universe

The Hubble parameter (H_0) is the constant of proportionality for the expression $v \propto D$ where v is the recession velocity of a galaxy and D is its distance from Earth. To find a value for H_0, we therefore need to plot a graph of v against D and find its gradient.

Part I: Analysing the absorption spectra to find the velocity of the galaxies

Each of the right-hand diagrams for the galaxies shows you:

1) (*at top and bottom*) an emission spectrum for Calcium (done on Earth, *i.e.* stationary), just like you have seen with a diffraction grating and spectrometer.
2) (*in between the emission spectra*) an absorption spectrum for the light from the relevant galaxy, taken as seen from Earth.

The b and c labels show you the positions of two lines in the Calcium spectrum, called the H and K lines. The wavelengths for all 7 of the lines are as follows:

Line	a	b	c	d	e	f	g
λ (nm)	388.87	396.47	402.62	414.38	447.15	471.31	501.57

The arrows beneath each absorption spectrum shows you the apparent redshift for the H and K lines, from the emission spectrum of Calcium as seen on Earth to the absorption spectrum for the galaxy as seen from Earth. Notice that they are all shifted to the right; towards the long-wavelength end of each spectrum, *i.e.* red-shifted. This confirms that each galaxy is moving away from us (receding).

To find the velocity of recession for each galaxy, you need to find out how far each of the H and K lines has been red-shifted and use:

$$\frac{\Delta\lambda}{\lambda} = \frac{v}{c}$$

…where $\Delta\lambda$ = the amount of redshift (in nm), λ = the original wavelength of the line (also in nm), v = the recession speed of the galaxy (in ms^{-1}) and c = the speed of light (in ms^{-1}).

1) Use the table of wavelengths above to work out a scale for the spectra (*i.e.* '1mm on the spectra = a red shift of … nm') and then use this to find Δλ in nm. You should expect that $\Delta\lambda$ (and hence v) should be the same for both the H and K lines, but double check that this is true on each spectrum. If it isn't, use an average value for $\Delta\lambda$. Then use the equation above to find v. Record your results in the table at the end of this sheet.

Part II: finding the distance from Earth to each galaxy

2) Measure the size of the 150″ scale at the bottom of the sheet to get a conversion factor for the pictures themselves (*i.e.* '1mm on the pictures = … seconds of arc'). Then measure the apparent width *w* of each galaxy in mm and convert this to an angle θ in arc-seconds. If the galaxy is elliptical, measure the maximum and minimum values of *w* and average them. Add your results to the table.

3) Now convert the angles from arc-seconds into radians (using the fact that 2π radians = 360°, and that 1° contains 60′ and 1′ contains 60″).

Converting this angle to a distance uses a technique similar to measuring the distance to stars using parallax. It relies on the assumption that the biggest galaxy in any given cluster is about 30 000 parsecs in diameter. We can confirm this for nearby galaxies whose distances can be measured by other means and, by using some simple trigonometry, we find that:

$$D\,(\text{in parsecs}) = \frac{30000}{\theta\,(\text{in radians})}$$

4) Use this rule to convert your angles in radians into distances in parsecs and then convert these into distances in metres (1 pc = 3.26 ly = 3.09 x 10^{16} m):

Galaxy in	$\Delta\lambda$ (nm)	v (ms^{-1})	w (mm)	θ (arcsec)	θ (rad)	D (pc)	D (m)
Virgo							
Ursa Major							
Corona Borealis							
Boötes							
Hydra							

Part III: finding H_0 from a graph of *v* against *D*

5) Plot a graph of *D* on the *x*-axis against *v* on the *y*-axis and the gradient of your graph will give you a value for the Hubble parameter (=H_0). Then you can calculate the age of the Universe $\left(\text{Age} \approx \frac{1}{H_0}\right)$.

How close is it to the accepted range of values? How accurate do you think your value is? How accurate are your measurements from the pictures?
For some *very* worthwhile practice, work out the uncertainty in your value for H_0.