Mass Defect, Binding Energy and Plotting the BE/Nucleon curve

In 1905, Einstein published several ground-breaking papers which changed Physics forever. One of these says that Energy and Mass were really two different forms of the same thing, related by 'the most famous equation in the world', $E = mc^2$ (where c = the speed of light).

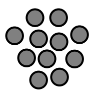
1) If this piece of paper has a mass of 5g, how much energy could you get from this amount of mass? Take $c = 3 \times 10^8$ ms^{-1}

2) If this piece of paper were suddenly transformed into that amount of energy, what do you think it would do?

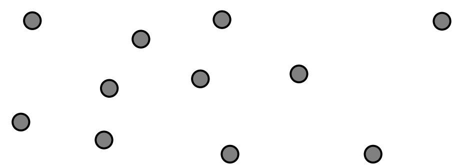
$E = mc^2$ also implies that, if you give an object some energy (*e.g.* potential energy) then, bizarrely, its mass increases:

3) By how much does the mass of a 1kg block increase when it is raised by 1 metre (*i.e.* is given GPE)?

It follows that, if you peel the electrons off an atom, you give each of them some EPE as you remove them from the atom (similar to the way that removing masses from the Earth gives them GPE). This increase in PE also happens if you separate all the protons and neutrons as well as removing the electrons. In other words, if you take an atom and separate all its constituent particles, then the separated particles have more PE than they did when they were bound together as a single atom:

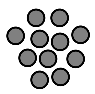


PE
(with nucleons together as a nucleus)

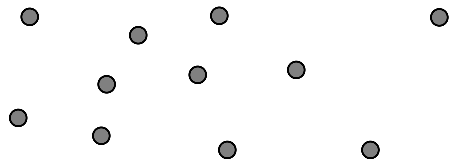


PE + **ΔPE**
(when the same nucleons have been separated)

We found out in 3) that an increase in PE results in an increase in mass, so this means that the separated particles must have also increased in mass as a result of being separated:



m
(with nucleons together as a nucleus)



m **+ Δm**
(when the same nucleons have been separated)

Effectively, the energy you use to separate the particles increases their mass. You can calculate this increase in mass (called the **mass defect**) using $\Delta E = \Delta mc^2$.

4a) Calculate the total mass of all the particles in a Helium-4 nucleus in atomic mass units, using the following:

Data: Mass of a proton = 1.00728 u (1u = one atomic mass unit)
Mass of a neutron = 1.00866 u

4b) The actual mass of a Helium-4 atom, measured as a whole, is actually 4.002603u, which is less than the value you have just calculated in 4a. However, this smaller value also includes the electrons. To find the actual mass of the nucleus alone, subtract the mass of the electrons.

Mass of an electron = 0.000548 u

4c) What, therefore, is the mass defect for Helium?

(*If you get 0.0304u, give yourself a pat on the back*)

5) Now work out how much energy this mass defect represents.
(*remember to convert the amu to kg first:* $1u = 1.660 \times 10^{-27}$ *kg*)

6) Now convert this to MeV. (1 eV = 1.602×10^{-19} J)

This figure (*you should have 28.3 MeV by now*) is the **binding energy** for Helium. If we're being fussy about it, we should say that it is **–**28.3 MeV. The – sign shows that this is the energy which is needed to be put *into* the system when the nucleons are separated.

In practice, there is a sneaky shortcut you can make when calculating the binding energy of an atom: you can convert the mass defect (in u) directly to energy (in MeV), rather than converting it into kg and then into MeV. To do this, we need to work out how many MeV is represented by 1u.

7) Using $E = mc^2$, show that 1u is the equivalent of 931.49 MeV.

[*To get an answer correct to 5sf, you must use raw data which is accurate to at least 5sf: $1u = 1.66054 \times 10^{-27}$ kg; $c = 2.99792 \times 10^{8}$ ms^{-1}; $e = 1.60218 \times 10^{-19}$ C*]

So, in summary:

Mass defect = (Mass of nucleons) – (Mass of the nucleus)

and

Binding energy = Mass defect x c^2
(J) (kg)

or

Binding energy = Mass defect × 931.49
(MeV) (u) (MeV u^{-1})

Plotting the Binding Energy per Nucleon curve

We can now use our new-found knowledge to plot a very useful graph which we'll discuss in more detail later. We want to plot Mass Number along the *x*-axis and Binding Energy ***per nucleon*** on the *y*-axis. For example:

> We worked out before that the binding energy for Helium was -28.3 MeV. As Helium contains 4 nucleons (remember that a nucleon is a general term meaning a proton or a neutron; *i.e.* a nucleon is something you find in a nucleus) then the binding energy *per nucleon* for Helium is -28.3 ÷ 4 = -7.07 MeV. You now have the first point to plot on your graph.

Draw your axes now, as follows:

- *x*-axis: Mass number A (going from 0 to 250)
- *y*-axis: Binding Energy per Nucleon (MeV) (going from 0 down to -9)

In the same way as we've done for Helium, now calculate the BE/nucleon for each of the following elements:

Element	Proton number (Z)	Mass[1] (u)	BE/nucleon (MeV)
^{1}H	1	1.007825	
^{2}H	1	2.014102	
^{4}He	2	4.002603	-7.07
^{7}Li	3	7.016004	
^{11}B	5	11.009305	
^{12}C	6	12	
^{14}N	7	14.003074	
^{16}O	8	15.994915	
^{20}Ne	10	19.9924356	
^{56}Fe	26	55.934934	
^{136}Ba	56	135.90456	
^{207}Pb	82	206.975903	
^{238}U	92	238.05082	

Plot all these on a graph and try to draw a curve through the points. It is not a wonderfully smooth curve by any means – data in nuclear physics rarely conforms to simple rules – but do what you can.

[1] Those of you with a good memory for numbers may spot that these values are almost but not quite the same as those listed as RAMs on the Periodic Table (*e.g.* Carbon on the Periodic Table has an RAM of 12.011). The numbers above are the *actual masses of the specific isotopes listed*, not abundance-weighted average masses of the substance as found on Earth, which is what is given on Periodic Tables for practical purposes.