Measuring Electron Diffraction and Confirming the de Broglie Relationship

The electron beam is diffracted as it passes through the thin carbon film sample. The central spot on the screen defines the 'straight through' direction. The beam is diffracted at an angle θ to this direction. Due to the polycrystalline nature of the carbon film, all directions will have a diffracted beam so, rather than a spot at the angle θ, we get a circle of diameter *D*.

Let the distance from the carbon film to the end of the diffraction tube where the fluorescent screen is located be *L*.

Thus the angle $\theta = \frac{\text{radius of the circle}}{L} = \frac{D}{2L}$ (1)

For a transmission grating, $n\lambda = s \sin\theta$

where n = the order of the diffraction ($n = 0$ is the undiffracted beam)
s = the grating spacing (in this case, the spacing of the planes of carbon atoms)

for small angles, $\sin\theta \sim \theta$, thus $n\lambda = s\theta$ (2)

substitute (1) into (2) to give: $n\lambda = \frac{sD}{2L}$

or: $\lambda = \frac{sD}{2Ln}$ (3)

But we still don't have a way of measuring λ (the wavelength of the electrons). De Broglie claims that $\lambda = \frac{h}{p}$ applies to electrons. To find the momentum p of an electron accelerated thought V volts, we use:

$$eV = \frac{1}{2}mv^2 = \frac{(mv)^2}{2m} = \frac{p^2}{2m}$$

thus $p^2 = 2meV$

So $\lambda = \frac{h}{p}$ becomes: $\lambda = \frac{h}{\sqrt{2meV}}$ (4)

Equating the two expressions for λ (equations (3) and (4)) gives:

$$\frac{sD}{2Ln} = \frac{h}{\sqrt{2meV}}$$

Now, since everything in this equation apart from D and V is a constant for a given set of apparatus, this means:

$$D \propto \frac{1}{\sqrt{V}} \text{ or } D\sqrt{V} = \text{Constant}$$

We can then vary V (the pd accelerating the electrons), measure the resulting D and see if $D\sqrt{V}$ is constant, at least within experimental uncertainty.

Due to the nature of the crystal structure of graphite, we actually get <u>two</u> concentric rings on the screen. For each of these, the grating spacing is different, so we need two sets of data, one for the inner circle (D_1) and one for the outer circle (D_2):

Accelerating pd V (kV)	**D_1 (mm)**	**D_2 (mm)**	**$D_1\sqrt{V}$ (kVmm$^{-1/2}$)**	**$D_2\sqrt{V}$ (kVmm$^{-1/2}$)**

If the two final columns yield more or less constant results, this means our analysis – which relied, in part, on $\lambda = \frac{h}{p}$ being true – was correct.