# Combining the gas laws

So far, from experiments, we have the following laws for gases:

Boyle's Law: $pV$ = constant (*if temperature and amount of gas are constant*)

Charles' Law: $\frac{V}{T}$ = constant (*if pressure and amount of gas are constant*)

Amonton's Law: $\frac{p}{T}$ = constant (*if volume and amount of gas are constant*)

These are often very useful, but the requirement for multiple quantities to be constant means we sometimes need more options. For example, in an engine, when the fuel vapour ignites and pushes the cylinder outwards, the hot gas will change in temperature, pressure and volume, so none of the above equations could be used.

We need to combine the three laws into one law which only requires the amount of gas to be constant. Here's how:

Firstly, the three constants referred to above do not have the same value, so let's call them $k_1$, $k_2$ and $k_3$:

$$pV = k_1 \qquad \frac{V}{T} = k_2 \qquad \frac{p}{T} = k_3$$

We can then multiply all of these expressions together to give:

$$(pV)\left(\frac{V}{T}\right)\left(\frac{p}{T}\right) = k_1 k_2 k_3$$

Which we simplify to:

$$\frac{p^2V^2}{T^2} = k_4$$ (where $k_4 = k_1k_2k_3$, just for simplicity)

Square root both sides to give:

$$\frac{pV}{T} = k_5$$ (where $k_5 = \sqrt{k_4}$, again for simplicity)

So we are left with:

$$\frac{pV}{T} = \text{constant}$$

This is called the **Combined Gas Equation**. Since p, V and T are included in this equation, they do not have to be constant so it is more generally useful than the individual gas laws we started with. The amount of gas, however, does need to be constant.

As with the three separate gas laws, a more useful form of the combined gas equation is $\frac{p_1V_1}{T_1} = \frac{p_2V_2}{T_2}$.