Log-Log Graphs

or, the nearest you'll get to magic with a graph

In the past, to check a mathematical relationship, you will often have plotted graphs. For example:

- If the pattern you are expecting to find is $V \propto I$, then you would plot V (on the *y*-axis) against I (on the *x*-axis) and the gradient would be equal to the constant of proportionality.
- If the pattern you are expecting to find is $p \propto \frac{1}{V}$, then you would plot p (on the *y*-axis) against $\frac{1}{V}$ (on the *x*-axis) and the gradient would be equal to the constant of proportionality.
- If the pattern you are expecting to find is $s \propto t^2$, then you would plot s (on the *y*-axis) against t^2 (on the *x*-axis) and the gradient would be equal to the constant of proportionality.

…and so on.

But what if you don't know what relationship to expect? What if you just suspect that $a \propto b^n$, where *a* and *b* are variables and *n* is an unknown constant? What graph could you plot? Well, you could try trial and error: plotting a against b^2, then a against b^3, then a against $\frac{1}{b}$ (*i.e.* n = -1), then a against $\sqrt{b}$ (*i.e.* $n = \frac{1}{2}$) and so on until you get a straight line, but it might take a while.

A much quicker method – that only requires one graph to be drawn – is to use logarithms, like this:

Start with what you suspect to be true:	$\boldsymbol{a \propto b^n}$
Which is the same as saying:	$\boldsymbol{a = kb^n}$ (where *k* is a constant)
Take logs[*] of both sides:	$\ln(\boldsymbol{a}) = \ln(\boldsymbol{k}) + \boldsymbol{n} \ln(\boldsymbol{b})$
Which is of the form:	$\boldsymbol{y = c + m x}$

So, a graph of ln(*a*) against ln(*b*) would be a straight line with the gradient equal to the power of *b* in the original relationship. So, if the gradient turned out to be 3, then you would know that $a \propto b^3$. If the gradient turned out to be -1.5 then you would know that $a \propto \frac{1}{\sqrt{b^3}}$. And so on. Also, if you wanted to know the constant of proportionality (*k*), it would be the antilog of the *y*-axis intercept on your graph.

These types of graphs are called log-log graphs[†] because you are plotting logarithms of numbers on each axis.

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[*] You may be surprised to see that natural logarithms (*i.e.* logarithms to the base *e*, where *e* = 2.71828…) are used. If you understand how logarithms work, you will appreciate that everything on this sheet could equally be done using logs to any base at all. However, when the base does matter – as it does in other areas of physics – it is nearly always natural logs which need to be used in Physics, because Physics is a description of the natural world. Physicists therefore tend always to use natural logs, even when the base is irrelevant. In fact, many Physicists would read the third equation above out loud as 'log *a* equals log *k* plus *n* log *b*'.

The number *e* crops up in so many areas of Physics and the natural world (radioactive decay, electric circuits, waves, seashells, snails and many more) that it gets a bit spooky, rather like π.

[†] Which a Physicist might well write as 'ln-ln graphs'.

Example: wobbly hacksaw blade

Here is a table of results for the time period (T) of a weighted hacksaw blade oscillating with a 100g mass at different lengths (L) from where the blade is clamped:

L (m)	T (s)		
0.147	0.380		
0.163	0.434		
0.177	0.481		
0.195	0.550		
0.204	0.587		
0.214	0.624		
0.232	0.732		
0.269	0.906		

It is thought that T and L are related by the equation $T = kL^n$.

By taking logs‡ of both sides, work out what sort of graph you could plot to give a straight line and find values for k and n:

> If I plot a graph of ……….. against ……….., then it should be a straight line with a gradient equal to ……….. and an intercept equal to ………..

Add headings and columns to the table above to do your calculations, then plot a graph and find values for *n* and *k*.

A note on logarithms and units

When you manipulate a quantity, the units usually change. For example, if you square a distance in metres, the result would have units of m^2. If you square root a resistance, the result would have units of $\Omega^{1/2}$ and so on. Mathematically, you can only take the logarithm of a pure number, *i.e.* something without units. This leaves you in a tricky situation when you want to take the logarithm of a time or a temperature, for example. The simplest way to deal with this is to write this: $\ln{(t/\mathrm{s})}$

This means you are taking the logarithm of the (time divided by seconds). Since time is measured in seconds, a time divided by seconds leaves a number with no units that you can then meaningfully take the log of. The placing of the brackets is vital!

This also means that the log of *any* quantity has no units.

More examples of correct presentation are **$\ln{(R/\Omega)}$** **$\ln{(Q/\mathrm{C})}$** **$\ln{(T/\mathrm{K})}$**

‡ You will use *natural* logs, naturally, because this is Physics.